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A Comparative Study of Multi-Attribute Continuous Double Auction Mechanisms

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Abstract
Auctions have been as a competitive method of buying and selling valuable or rare items for a long time. Single-sided auctions in which participants negotiate on a single attribute (e.g. price) are very popular. Double auctions and negotiation on multiple attributes create more advantages compared to single-sided and single-attribute auctions. Nonetheless, this adds the complexity of the auction. Any auction mechanism needs to be budget balanced, Pareto optimal, individually rational, and coalition-proof. Satisfying all these properties is not so much trivial so that no multi-attribute double auction mechanism could address all these limitations. This research analyzes and compares the GM, timestamp-based and social-welfare maximization mechanisms for multi-attribute double auctions. The analysis of the simulation results shows that the algorithm proposed by Gimple and Makio satisfies more properties compared to other methods for such an auction mechanism. This multi-attribute double auction mechanism is based on game theory and behaves fairer in matching and arbitration.

Keywords: Double auction, Multi-attribute auction, Continuous multi-attribute double auction, Coalition-proofness

1. Introduction
Auctions have been a competitive method of buying and selling valuable or rare items for a long time (Fasli, 2007). Recently and in addition to electronic commerce, pricing and allocation of goods and services through auctions has been widely employed

over grids, clouds, and networks in different types of applications. Meanwhile, multi-attribute single-sided (reverse) auctions in which several other attributes are being negotiated besides the price between a seller (buyer) and many buyers (sellers) are becoming very demanding. Auctions have procedures for matching the offers. The auction ends with a settlement and finalizing the deal; the buyer takes possession of the item and the seller receives the money.

In a multi-attribute double auction, many sellers negotiate with many buyers over many attributes. Despite of single-attribute single-sided auctions, determining buyers and sellers who win the deal in a multi-attribute double auction is not straightforward. Each buy (sell) bid can be matched against several sell (buy) bids on the other side of the market. Therefore, complex computations and a set of arbitration rules are required to determine the final deals which are affected by the values the buyers and sellers submitted for each attribute. Determining the winners and admissible combination of values for the trade attributes are two key issues in the auction mechanism.

Winner determination methods in multi-attribute double auctions have been based on timestamp or for welfare maximization. An auction must possess specific properties to operate profitable while encourages the users to participate in. A proposed method by Gimpel et al, (Gimpel & Makio, 2006) pays special attention to the properties of a mechanism. In this study, we analyze and then implement compare the methods through computational simulations.

The paper is organized as follows. In Section 2, we overview a few concepts before reviewing the mechanisms proposed for multi-attribute double auctions in Section 3. The methods are implemented and evaluated in Section 4. Section 5 concludes the study.

2. General Concepts

This section describes required properties of a multi-attribute double auction and the concept of the utility function which participants use to show their preferences.

2.1 Required Properties for an Auction Mechanism

The properties required for an auction are as follows (Gimpel & Makio, 2006; Weiss 1999; Schnizler, 2008):

Budget-balance: This property guarantees the amount of money entering the auction (by participants) equals the amount exiting it (to participants). In a weak budget-balance mechanism, the sum of the entering and exiting money must be non-negative; that means the auction will not run at a loss.

Pareto optimality: A solution is Pareto optimal if both sides of the auction are satisfied and no one could be made better off without making someone else worse off.

Individual rationality: This property shows that the profit gained by participating the auction must not be less than not participating it.

Coalition-proofness: This property states that a group of traders must not gain more profit by doing a coalition outside the auction mechanism.

2.2 Preferences of Participants

Preferences of an agent (buyer/seller) in the auction can be expressed using a utility function (U). If the agent prefers offer O over O’, then O has a higher utility than O. If U(O) = U(O’), the agent is indifferent between the offers (Schnizler, 2008).

The agent assigns utility values between 0 and 1 to the alternatives; 1 to the highest priority and 0 to the lowest priority. When an option is multi-dimensional, each dimension may be of a different importance for the bidder. A weighted additive multi-attribute utility function simply sums up the weighted utilities of individual factors, so that the weights are normalized and their summation is 1 (Weiss, 1999).

3. Multi-Attribute Double Auction Mechanisms

In a multi-attribute double auction mechanism many buyers and many sellers in opposite sides of the market tend to decrease and increase the bids, respectively. Therefore, determining the winners and the value of the deal is not trivial and calculations are required to be performed (Gimpel & Makio, 2006; Wurman, Walsh, & Wellman, 1998):

1- When more than one bid can be matched with one opposite bid, which one must be selected? In other words, who are the winners?

2- When more than one combination of values of attributes (e.g. price, guarantee, and delivery date) are acceptable, which combination is selected as the final deal?

For addressing these questions, three multi-attribute double auction mechanisms are considered in this section.

3.1 GM Algorithm

In the algorithm proposed by Gimpel and Makio (2006), winner determination and choosing the attribute values of the deal is through matching and arbitration phases. The algorithm considers these phases in single and multi-attribute continuous double auctions.

3.1.1 GM Algorithm in Single-Attribute Continuous Double Auctions

In a continuous single-attribute double auction, there is a spread of bids to buy or sell. Buy bid and sell bid are said ‘bid’ and ‘ask’, respectively. Figure 1 shows a set of such bids. Bids B_1, B_2, and B_3 by the buyers and asks A_1, A_2, and A_3 by the sellers are placed. Each bid has a specific price range. For example, A_3 shows that the seller wants to sell the item at a price of 100 or above, while B_3 shows that the buyer wants to buy the item for 110 or lower price.

In call markets, all bids are collected and checked at the end of a period to determine the winners. However, in a continuous double auction, winners are determined once a new bid or ask arrives. When a sell (buy) bid enters, matching phase looks for existing buy (sell) bid which overlaps the new arrival bid.

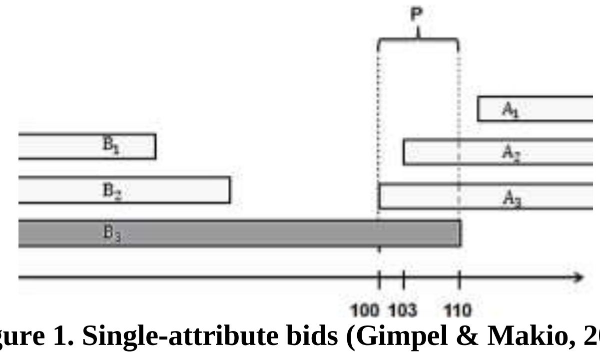

Figure 1. Single-attribute bids (Gimpel & Makio, 2006)

3.1.1.1 GM Matching in Single-Attribute Continuous Double Auctions

Matching is a process that determines only one bid overlapping with a new bid. In Figure 1, B_3 can be matched with A_2 or A_3. Referring a prevalent rule in financial markets (which are usually double sided markets), the new bid (ask) is matched with an ask (bid) with the lowest (highest) price. Therefore, B_3 is matched with A_3, because A_3 has a smaller lower bound than all other sell bids that overlap with B_3.

3.1.1.2 GM Arbitration in Single-Attribute Continuous Double Auctions

At the arbitration phase, an exact deal (i.e., the price) between matching bids is determined. Bids are often matched in a price interval not a specific and exact price. In Figure 1, for example, any price in the interval P=[100, 110] guaranties the properties of individual rationality, Pareto optimality, and budget-balance for A_3 and B_3. This interval and only this interval includes all possible solutions which leads to profit and no loss for both sides.

It is worth to consider the lack of coalition-proofness in prices above 103 (p>103). As shown in Figure 1, A_2 also overlaps with B_3. However, since it has a smaller overlapping interval P=[103,110], A_2 is less flexible in increasing profit for itself and B_3. Due to less flexibility, it is in higher chances of coalition and conducting a deal outside the auction. To achieve coalition-proofness, the interval where A_2 and B_3 overlap is subtracted from the initial price interval P. Therefore, any price p at new interval P=[100, 103] is fair and the best price for the deal (Gimpel & Makio, 2006).

3.1.2 GM Algorithm in Multi-Attribute Continuous Double Auctions

As price is not the only important issue in buying or selling an item, multi-attribute auctions, where more than one issue is considered are very common now (Fasli, 2007). This section extends the issues discussed in Section 3.1.1 to multi-attribute double auctions.

3.1.2.1 GM Matching in Multi-Attribute Continuous Double Auctions

Figure 2 shows an example of a multi-attribute continuous double auction over two attributes (price and delivery). Three buy bids B_1, B_2, and B_3 are already in the system when the sell bid A enters. Every bid in this example is shown with a two dimensional rectangular space which is a subset of utility values of possible agreements. The seller bids A to sell his item at a price above 100 with a delivery of 30 to 105 days. These bidders have different preferences and utilities over different trades. Preferences of seller A are shown through three curves. While the seller prefers higher price or longer delivery (the upper right corner), these buyers accepts a lower price or shorter delivery. The highest curve (i.e.) is the best and the most profitable for A ($U_1<U_2<U_3$).

In a single-attribute continuous double auction, the new bid is matched with the bid that maximizes its utility. In Figure 1, the sell bid with the lowest price was chosen because it maximized the buyer's utility. Similarly, in multi-attribute continuous double auction of Figure 2, B_2 is selected in the matching phase, because bid A has a maximum utility with B_2. The result of the matching phase is the selection of a pair (A, B*), so that

$$B^* = \arg\max_{B\in\{B_1,\ldots,B_n\}} \max_{x\in A\cap B} U_a(X) \quad (1)$$

Where, U_a is the multi-attribute utility function for the bid A and is a way to express the agents' (buyers or seller) preferences (Fasli, 2007) and X is a point in the space of possible agreements. A is the new bid and B_1, B_2… B_n are the bids already in the system.

In the matching phase, only the utility function of the bid A, the new bid, is used and the utility functions of other existing bids are irrelevant to the auction mechanism B*'s utility function is only considered later in the arbitration phase (Gimpel & Makio, 2006).

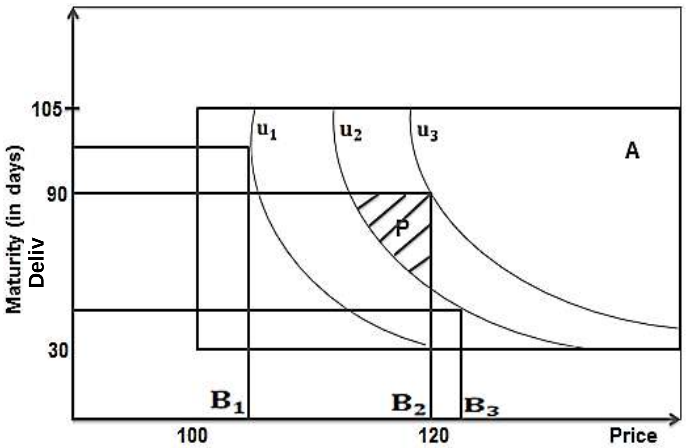

Figure 2. The matching stage in two-attribute bids (Gimpel & Makio, 2006)

3.1.2.2 GM Arbitration in Multi-Attribute Continuous Double Auctions

After choosing the pair (A, B*) by matching phase in single-attribute continuous double auction, the arbitration phase determines the deal for the transaction. In Figure 2, since B_2 was selected in matching phase, any deal with the price between 100 and 120 and delivery time of 30 to 90 days has the properties of individual rationality and budget balance.

Although these price and delivery values guarantees holding these two properties, there is a possibility of coalition. B_1 and B_3 have a higher chance of forming a coalition (e.g., for prices below 100 and deliveries shorter than 30 days) because they are less flexible in increasing A's utility. The final deal must maximize the utility of A so that it cannot find a better deal outside the auction. To achieve coalition-proofness, a bid with the maximum utility is then chosen among the overlapping bids that were not selected in the matching phase. The area under its curve is then eliminated. In this shaded area P, all the properties of individual rationality, budget balance and coalition-proofness are held.

As shown in Figure 3, the area P is mapped to the utility space. The auction mechanism must choose Pareto optimal solutions (i.e. the upper right border of the utility space). These points maximizes the seller and the buyer's utilities and eliminates any conflicts.

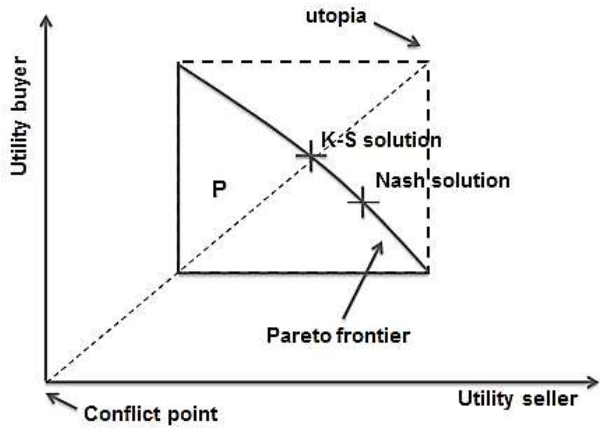

Figure 3. Aribitration phase in the utility space (Gimpel & Makio, 2006)

Kalai-Smordnisky and Nash solutions are the most famous solutions that use game theory to solve such problems. The solutions are based on conflict and utopia points. If the two bidders will not reach an agreement, a no trade conflict point is gained. Utopia point is where the two bidders have maximum utility.

The Nash solution determines the deal which maximizes the product of the difference between the utility of each side and the utility at conflict point (Systems Analysis Laboratory, 2012).

Utility at conflict point D = (d_s, d_b)

N(P) $=p^*, p^* = \arg\max_{\mathrm{p}\in\mathrm{P}} (U_s(\mathrm{p}) - \mathrm{d}_s) \times (U_b(\mathrm{p}) - \mathrm{d}_\mathrm{b})$ (2)

The Kalai-Smordnisky solution is the intersection point of the negoation space border and the line between the conflict and utopia points. This solution always assumes the same utility for the best solution (the Kalai-Smordnisky solution) for both sides of the negotiation.

For the pair (A,B*) chosen in the matching phase, the arbitration phase selects a deal x* ∊ A∩B*. In x* the *individual rationality* and *budget balance* properties are guaranteed. Moreover, if $\mathrm{B}' = \arg max_{B\in\{B_1,..,B_n\}/B^*} max_{x\in A\cap B} U_A(\mathrm{x})$, then $\mathrm{U}_A(x^*) \geq \mathrm{U}_\mathrm{A}(\mathrm{B}')$. That is, the priority of B' is less than of x*. Therefore, no deal outside the auction mechanism can bring a higher utility for the bidders. This gurantees *coalition-proofness*. In addition, x* is *Pareto optimal*, because there is no other solution in which one side can gain more without making the other side achieves worse.

3.2 Timestamp-based Algorithm

Similar to GM, this algorithm also has two matching and arbitration phases. However, its arbitration phase considers the arrival time of bids besides other attribute values in the multi-attribute continuous double auction.

The matching phase, chooses the earliest bid in the opposite side of the market which ovelaps the new bid. To determine the deal between the matched pair, the arbitration phase uses the mean utilities (Berśeus, 2007). Exapmles presented in Section 4.1 exhibits the approach.

3.3 Social-Welfare Maximization Algorithm

The algorithm is demonstrated through the following example. Three bids A, B, and C exists when X enters the system. For instance, X is a sell bid and types of

A, B, and C are assumed opposite of X. Every bidder has a utility for every possible attributes (price, delivery time, and color). In this example, X has the utilities 0.8 and 0.2 for the prices 100 and 110, 0.48 and 0.52 for deliveries of 30 and 60 days, and 0.51 and 0.49 for the colors red and blue, respectively. It is worth to mention that (1) the sum of all utilities for every attribute is 1, and (2) weights of all attributes are assumed equal (Gimpel & Makio, 2006).

Table 1. An example of the social welfare algorithm (Gimpel & Makio, 2006)

Order	Color		Price		Delivery	
	blue	Red	110	100	60 Days	30 Days
A	0.55	0.45	0.62	0.38	0.42	0.58
B	0.43	0.57	0.33	0.67	0.39	0.61
C	0.11	0.89	0.30	0.70	0.99	0.01
X	0.51	0.49	0.80	0.20	0.48	0.52

For each attribute value, the algorithm determines the maximum utility summation by calculating sum of each individual utility of the new bid against every existing bid in the opposite side. This concludes both the matched pair and the deal.

For example, calculations related to the color attribute for X and A are as follows:

$\text{Max}\ \{(U^{X}_{color=red} + U^{A}_{color=red}), (U^{X}_{color=blue} + U^{A}_{color=blue})\}$
$= \text{Max}\ \{(0.49+0.45), (0.51+0.55)\} = \text{Max}\{0.94, 1.06\} = 1.06.$

This means that the color blue results in a higher utility in the color attribute for X and A. This calculation is repaeted for attributes price and delivey between X and A:

(X, A):

$\text{Max}\ \{(U^{X}_{color=red}+U^{C}_{color=red}), \mathbf{(U^{X}_{color=blue}+U^{C}_{color=blue})}\} +$
$\text{Max}\ \{(U^{X}_{price=100}+U^{C}_{price=100}), \mathbf{(U^{X}_{price=110}+U^{C}_{price=110})}\} +$
$\text{Max}\ \{\mathbf{(U^{X}_{delivery=30}+U^{C}_{delivery=30})}, (U^{X}_{delivery=60}+U^{C}_{delivery=60})\}$

= Max {(0.49+0.45), **(0.51+0.55)**}
+ Max {(0.20+0.38), **(0.80+0.62)**}
+ Max {**(0.52+0.58)**, (0.48+0.42)}
= 3.58

After determining the most profitable attribute values between each pair of bids, we have:

(X→A): U{color=blue, price=110, delivery=30}=3.58,
(X→B): U{color=red, price=110, delivery=30}=3.32,
(X→**C**): U{color=**red**, price=**110**, delivery=**60**}=**3.95**.

Therefore, a pair with a maximum summation, here X and C, is then chosen. And, in the arbitration phase, the values that play a role in maximizing the utility of attributes for this selected pair are chosen (here, “red” for color, “110” for price and “60 days” for delivery).

If more than one solution exists, the earliest matching bid is selected.

4. Evaluation

Not all multi-attribute continuous double auction mechanisms introduced in Section 3, satisfy all required properties for an auction. The GM algorithm proposed by Gimpel et al. (see Section 3.1) guarantees all needed properties (i.e., individual rationality, budget balance, Pareto optimality, and coalion-proofness). Coalition-proofness is very important in the arbitration phase in determining the best transaction. However, the timestamp-based algorithm (see Section 3.2) is sometimes not as efficient as GM algorithm and achieves differently in the arbitration phase. In this section, the mechanisms are evaluated.

4.1 GM vs. Timestamp-based Algorithm

In this section, GM and timestamp-bsed algorithms are evaluated respecting the properties necessary for a multi-attribute continuous double auction in either discrete or continuous value sapces.

4.1.1 GM and Timestamp-based Algorithms in Continuous Value Space

An example of a continuous value space is given in Figure 4. First, A_0, A_1, A_2, A_3, and A_4 have been entered and intend to sell goods and B_1 has been the last bid entered intending to buy goods.

In the matching phase of timestamp algorithm, A_1 is selected among the exisiting bids that overlap B_1, because it is earliest. The intersection of A_1 and B_1 is P=[100, 110]. The midpoint of this interval, C = (100+110)/2 = 105, is chosen at the arbitration phase.

Matching phase in GM algorithm chooses A_3 and the intersection of B1 and A3 is the inteval M = [96, 110]. To achieve coaltion-proofness in the arbitration phase, [100, 110] is is elminated from the interval M and point D is selected for the deal according to the Nash solution (Equation 2); because the product of utilities of B_1 and A_3 at this point is higher than of other points in [96, 100] (e.g., E or F). Then, this encourages the agents gain higher profits by participating the auction and satisfies individual rationality.

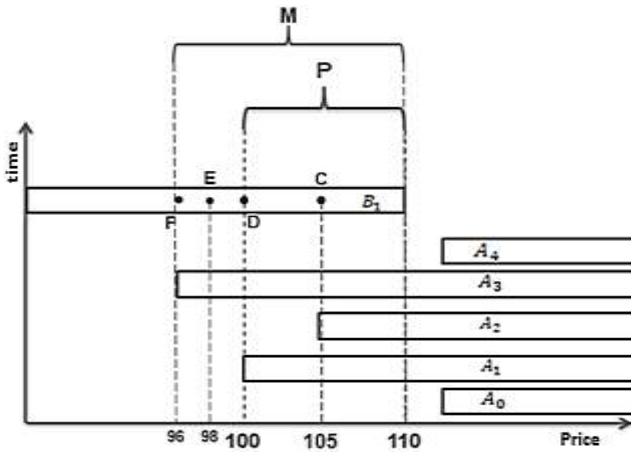

Figure 4. The chronological bids in a single-attribute continuous double auction over continuous value space.

A_1 and A_3 and points C and D are respectively the results of the matching and arbitration phases in timestamp and GM algorithms. The results summarized in Figure 5, where the arrows describe the range of the profits can be gained by each bidder. For instance, B_1 and A3 respectively gains a profit of 10 and 4 at point D; a total profit of 14 units. The profits at point C are respectively 5 and 5 for B1 and A3; giving a total of 10 units.

A solution is considered Pareto optimal if it satisfies both sides of the negotiation and there is no other solution that can increase the satisfaction of one side without making the other side worse off. The timestamp algorithm is not Pareto optimal, as there is a point D in which both sides are more satisfied than in point C.

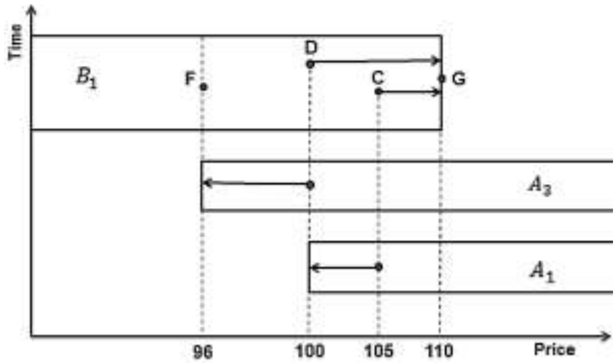

Figure 5. Summary results of the example presented in Figure 4.

4.1.2 GM and Timestamp-based Algorithms in Discrete Value Space

Assuming two attributes price and delivery for the items in this auction, suppose prices of 100, 110, and 120 and deliveries of 20 and 30 days are available, for example. As shown in Table 2, bidder B intends to buy goods and bidders S and T intend to sell. Bidders S and T are already in the system when bidder B enters. The table shows the utilities of bidders based on their own utility functions in each combination of attribute values in the space of possible discrete multi-attribute agreements. The bigger the utilty, the higher the preference, and vice versa. The only possible deal for seller T, is selling the item in the price of 100 and delivery of 30 days, gaining a utility of 1.0. For the buyer B and seller S, there are respectively 3 and 6 possible deals. The highest price (120) and the longest delivery (30 days) lead to the highest utility (0.96) for the new comer B.

Table 2. Bidders' utility (adapted from Gimpel & Makio, 2006)

Trader	Price			Delivery
	100	110	120	
B (buyer), Arrival: 6	0.16	0.48	0.80	20 days
	0.32	**0.64**	0.96	30 days
S (seller), Arrival: 4	0.50	-	-	20 days
	0.46	0.16	-	30 days
T (seller), Arrival: 2	-	-	-	20 days
	1.00	-	-	30 days

Once B enters, the system checks to see if it overlaps with any bids from the opposite side. In this example, B overlaps with both S and T. After the matching phase, a special combination of attributes must be determined for the two matched bidders.

In the matching phase, a sell bid that maximizes B's utility is determined. Among all points in the possible agreement space for B (3 points with S and only one points with T), a highest utility point that overlaps with a sell bid is in price of 110 and delivery of 30 with S. This approach sometimes creates an opportunity for coalition.

The aim of the arbitration is in fact maximizing the profit of the new bid which has more bargaining power in the auction. The method to reach a deal for B which satisfies required auction properties is as followis:

- For B, the bids in price of 120 or in price of 110 and delivery of 20 days does not hold individual rationaloty. Thus, these combinations are eliminated.
- The three remaining combinations in price of 100 and delivery of 20 or 30 days as well as in price of 110 and delivery of 20 days satisfies Pareto optimality and budget balance. This means that the auction mechanism runs without any loss for these combinations and there is no other solution for the bidders that generates a higher profit.
- Coalition-proofness does not hold for the price of 100 and delivery of 20 days. In addition, the bid in the intersection of S and T with the price of 110 and delivery of 30 days maximizes the profit. Furthermore, because T has a lower utility than S for that bid and is then less flexible in increasing the utility of B, there is a chance of coalition outside the auction mechanism for T.

To achieve coalition-proofness, sum of the utility of individual bids corresponding to B and S combination is calculated and the combination with lowest total utility is eliminated (Gimpel & Makio, 2006). The utility of the combinations of bids B and S is as follows:

$\mathrm{U}^{\mathrm{S}}_{\mathrm{delivery=20,price=100}} + \mathrm{U}^{\mathrm{B}}_{\mathrm{delivery=20,price=100}} = 0.50 + 0.16 = \underline{0.66}$

$\mathrm{U}^{\mathrm{S}}_{\mathrm{delivery=30,price=100}} + \mathrm{U}^{\mathrm{B}}_{\mathrm{delivery=30,price=100}} = 0.46 + 0.32 = 0.78$

$\mathrm{U}^{\mathrm{S}}_{\mathrm{delivery=30,price=110}} + \mathrm{U}^{\mathrm{B}}_{\mathrm{delivery=30,price=110}} = 0.16 + 0.64 = 0.80$

And since B and T ovelap, only the bid with the price of 100 and delivery time of 20 days is eliminated to achieve coalition-proofness. Thus, only two bids in prices of 100 and 110 and delivery of 30 days wil remain. Now, using the Nash solution in cooperative game theory (Equation 2), the combination with the maximum product is selected.

$U^{\mathrm{B}}_{\mathrm{delivery=20,price=100}} \times U^{\mathrm{S}}_{\mathrm{delivery=20,price=100}} = 0.46 \times 0.32 = \mathbf{0.1472}$

$U^{\mathrm{B}}_{\mathrm{delivery=30,price=110}} \times U^{\mathrm{S}}_{\mathrm{delivery=30,price=110}} = 0.16 \times 0.64 = 0.1024$

Max(0.1472 , 0.1024) = 0.1472

The deal with maximum profit for B and S has a price of 100 and delivery of 30 days.

In the matching phase of the timestamp alorithm, T will be selected with respect to the arrival time of the bids and results in a deal with the price of 100 and delivery me of 30 days.

Individual rationality of the deal is measured using the difference between the minimum utility of the bidder and the utility gained in the arbitration phase. Consequently, the bidders' satisfaction is calculated as the individual rationality for the two winners (see Table 3).

Table 3. Individual rationality and satisfaction of each bidder in GM and Timestamp mechanisms

Mechanism	Individual rationality		Satisfaction of bidders
	Seller	Buyer	
GM	0.16 - 0.16 = 0.0	0.64 - 0.16 = 0.48	0.48 + 0.0 = 0.48
Timestamp	1.0 - 1.0 = 0.0	0.32 - 0.16 = 0.16	0.16 + 0.0 = 0.16

To compare Pareto optimality of timestamp and GM algorithms, the difference between the minimum utility and the utility obtained in the arbitration phase for the new bid is calculated (Table 4). For this property to hold, there should be no other solution that can increase the bidders' satisfaction.

Table 4. Pareto optimality for the new bid (B) in GM and Timestamp-based mechanisms

GM	Timestamp
0.64-0.16=0.48	0.32-0.16=0.16

It can be seen that GM algorithm guarantees individual rationality and Pareto optimality significantly better than the timestamp algorithm.

The simulation results of implementing the two algorithms for 100 randomly generated bids in a continuous multi-attribute continuous double auction, as shown in Figure 6, also demonstrates that both individual rationality and Pareto optimality are significantly better satisfied in GM algorithm.

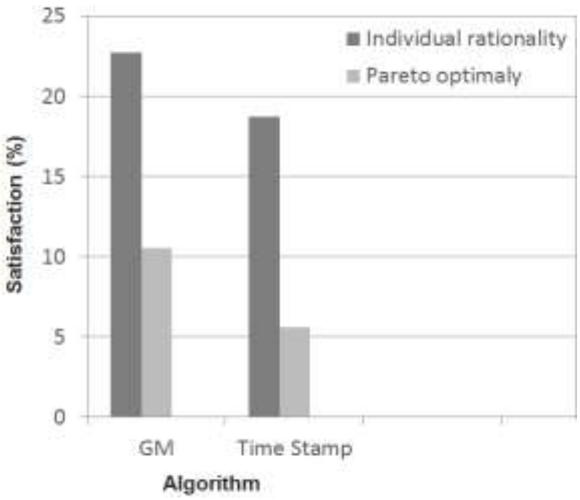

Figure 6. Comparing individual rationality and Pareto optimality in simulation results of GM and Timestamp-based mechanisms

4.2 Welfare-Maximization vs. GM Algorithm

In sosial welfare algorithm as mentioned in section 3.3, every bidder describes the utility for all the values of each individual attribute so that sum of the utilities for each attribute is 1 (e.g., utilities of 0.58 and 0.42 respectively for deliveries of 30 and 60 days in Table 1).

In GM algorithm (Gimpel & Makio, 2006), bids are described through the utility assigned to a combination of attribute values which are not necessarily reported for all attributes. That is, there may be no values for a few attributes in the package bid.

In the social welfare algorithm, on the other hand, a utility is assigned to every attribute per each value and the bidder should consider this complete information prior to participating the auction.

After finding a bid in the system that forms the highest profit in matching the new bid (matching phase), the attribute values that makes the highest utility will be determined (arbitration). For the sake of simplicity, the problem is explained in a conituous single-attribute example. It can be extended to discret multi-attribute continuous double auctions.

As shown in Figure 7, we assume that A_2 and A_3 are already in the system intending to sell and B_3 enters the system intending to buy. B_3 plans to buy goods for up to 110 monetary units, and A_2 and A_3 start selling at 100 and 103, respectively. We also assume three points C, D, and E in the intersection of these bids to consider the surplus of these points for each bidder. In C for instance, the surplus for B_3 is zero because the purchase is done at the highest possible price. The surplus for A_3 is 10 at this point, because the minimum price is 100. This value is 7 for A_2. Other values are also calculated at each point.

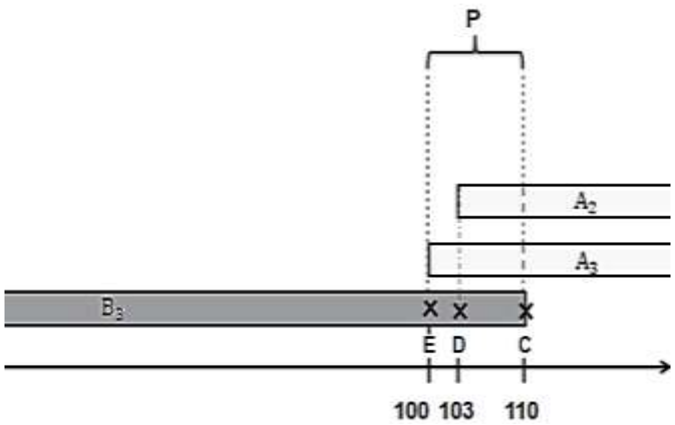

Figure 7. Bids in a single-attribute continuous double auction over continuous value space

In the social welfare algorithm, we also calculate the sum of surpluses (S) for the new bid as well as all other bids in the system.

In point C, where the price is 110:

S_{B_3} = 0, S_{A_3} = 110 -100 = 10, S_{A_2} = 110 - 103 = 7,

sum of surpluses of A_3 and $B_3 = S_{B_3} + S_{A_3}$ = 0 + 10 = **10**,

sum of surpluses of A_2 and $B_3 = S_{B_3} + S_{A_2}$ = 0 + 7 = 7.

In point D, where the price is 103:

S_{B_3}= 110 – 103 = 7, S_{A_3}= 103 – 100 = 0 , S_{A_2}= 0,

sum of surpluses of A_3 and $B_3 = S_{B_3} + S_{A_3}$ = 7 + 3 = **10**,

sum of surpluses of A_2 and $B_3 = S_{B_3} + S_{A_2}$ = 7 +0 = 7.

In point E, where the price is 100:

S_{B_3}= 110 – 100 = 10, S_{A_3}= 0, S_{A_2}= 100 – 103 = -3,

sum of surpluses of A_3 and $B_3 = S_{B_3} + S_{A_3}$ = 10 + 0 = **10**,

sum of surpluses of A_2 and $B_3 = S_{B_3} + S_{A_2}$ = 10 – 3 = 7.

It is observed that the combination of A_3 and B_3 leads to the maximum profit.

Thus, the result of matching phase is A_3 and B_3. In the arbitration phase, the point that maximizes the sum of surpluses is selected. However, the surplus is the same in all three points and therefore the result of the arbitration phase is all the points within the interval [100, 110].

According to GM algorithm the resulting interval is [100, 103]. In this interval, there is a no chance of coalition, because A_2 is less flexible to make a profit for B_3.

However, social welfare algorithm suffers a possible coalition, as the interval [103, 110] can not guarantee coalition-proofness propertiy for the mechanism.

5. Conclusions

An auction mechanism needs to have properties which not only allow it to be run without any loss but also make the traders incentive to buy and sell items by participating the auction mechanism. These properties are: budget balance, Pareto optimality, individual rationality and coalition-proofness. GM, Timestap-based, and social-welfare maximization algorithms was simulated and compared in multi-attribute continouse double auction mechanism. The fairest algorithm is GM proposed by Gimpel et al. Although, there are situations in which the Pareto optimality of GM is not as strong as the optimality of the social welfare algorithm, it is the only solution that is coalition-proof. It however, is more individually rational and Pareto optimal in average and is then considered as an optimal mechanism for multi-attribute continuous double auctions.

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